UNCLASSIFIED

AD 245 717

Reproduced by the

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

UNCLASSIFIED

AID Report 60-67 6 October 1960

ITEM OF INTEREST

Prepared by

Science and Technology Section
Air Information Division

SUBJECT: A New Theory of Symmetry

SOURCE : Shubnikov, A. V. Similarity symmetry (Preliminary report). Kristallografiya, v. 5, no. 4, Jul-Aug 1960, 489-496. QD901.K7, v.5

The author attempts to give a mathematical description of some operations of a special type of symmetry called similarity symmetry.

Similarity symmetry is understood as an extension of classical symmetry, which is based on the concept of two equalities, direct congruity and inverse congruity. In similarity symmetry the concept of equality is extended to parts of a figure which are similar geometrically without being equal in area or volume. A symmetrical position of the given parts of a figure is achieved by multiplication of any initial parts (or part), using the operations described below. These operations place similarly shaped parts into definite positions, forming the whole figure.

Shubnikov enumerates the following new operations of this symmetry:

a) Operation K consists in a parallel translation of similar parts with a simultaneous change of their size according to a given scale. The point of intersection of straight lines drawn through corresponding points of the figure parts, is called the special point of the figure. Figure 1 shows an example of the similarity symmetry K.

Fig. 1. Similarity Symmetry K

Fig. 2. Similarity Symmetry L

Fig. 3. Similarity Symmetry L

b) Operation L consists in a rotation of a part about a fixed axis through a given angle followed by a translation of the part, as in Operation K. (See Figures 2, 3, 4, 5, and 6). L can be defined as a <u>spiral motion</u> around the similarity axis L.

c) Operation M is analogous to the operation of simple or sliding mirror reflection in classical symmetry. It can be called a mirror reflection in the plane of similarity. The method of operating can be seen from Figures 7, 8, and 9. The bisector of the angle A'OA (Figure 7) indicates the position of the plane of similarity.

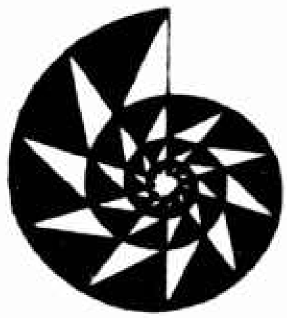

Fig. 4
Similarity Symmetry L

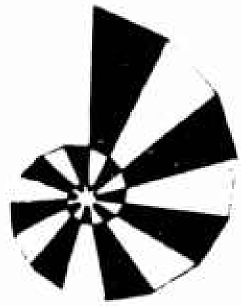

Fig. 5
Similarity Symmetry L

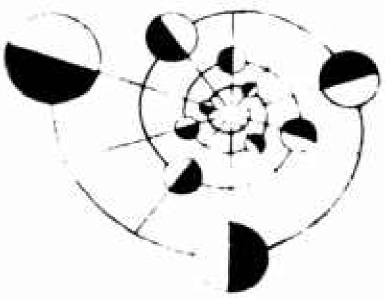

Fig. 6
Similarity Symmetry L

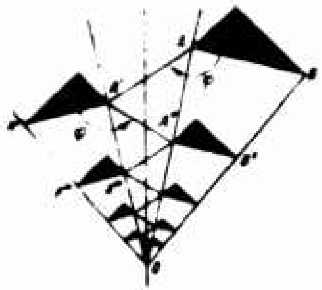

Fig. 7
Similarity Symmetry M

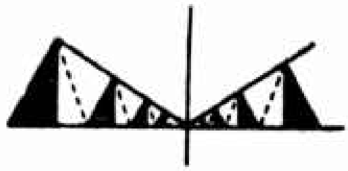

Fig. 8. Similarity Symmetry M

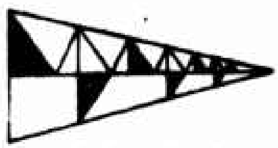

Fig. 9. Similarity Symmetry M

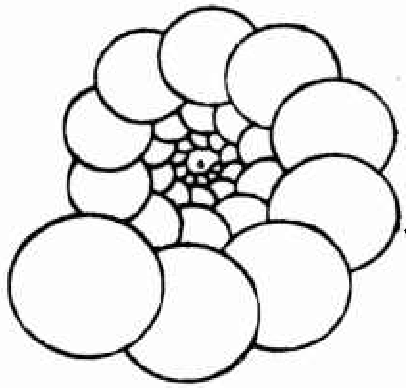

Fig. 10. Similarity Symmetry N

The operations K, L, and M represent all conceivable operations for similarity symmetry in a plane. These operations can be combined with the operations of classical symmetry, as discussed below. In this preliminary article, Shubnikov gives only one operation (Operation N) applicable to three-dimensional figures. (Figure 10). Operation N can be considered as a multiplication of Operation L by Operation K.

If the similarity axis L coincides with a symmetry axis of any order, similarity groups, designated as 2·L, 3·L, 4·L, etc., arise. (Figures 11, 12, 13). A symmetry axis and a similarity plane parallel to it produce similarity groups 2·M, 3·M, 4·M, etc. (Figure 14). Finally, three elements of similarity symmetry, namely, one of the similarity elements K, L, or M, a symmetry axis and a symmetry plane m give rise to the groups:

mL, 2·mL, 3·mL, etc., or mM, 2·mM, 3·mM, etc.

Some examples of the figures arising from such combinations are shown in Figure 15, 16, 17, and 18.

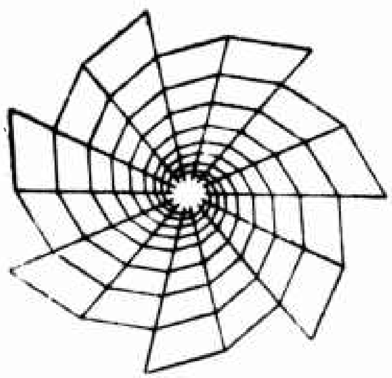

Fig. 11
Similarity Symmetry 7·L

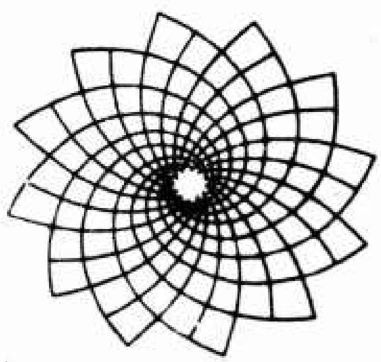

Fig. 12
Similarity Symmetry 12·L

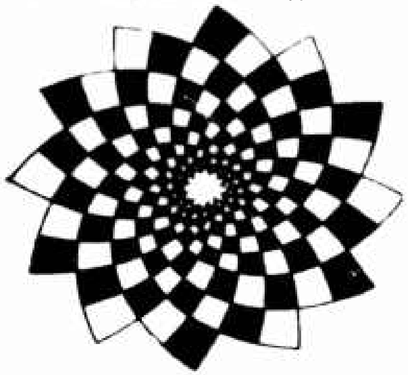

Fig. 13
Similarity Symmetry 6·L

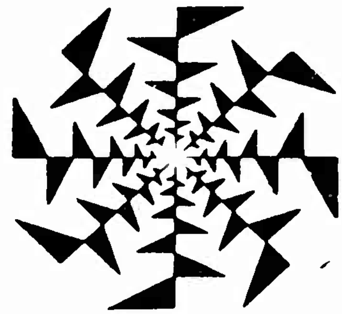

Fig. 14
Similarity Symmetry 8·M

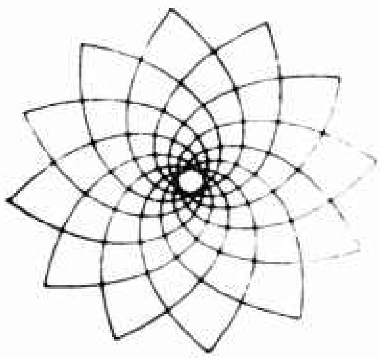

Fig. 15
Similarity Symmetry 12·mL

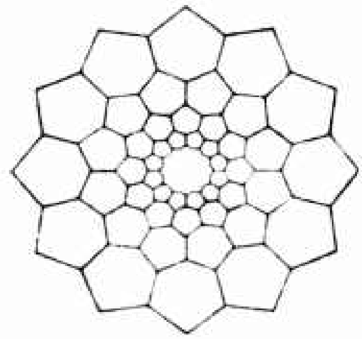

Fig. 16
Similarity Symmetry 12·mL

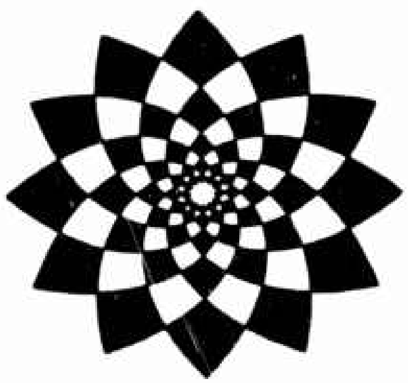

Fig. 17
Similarity Symmetry 2·mK

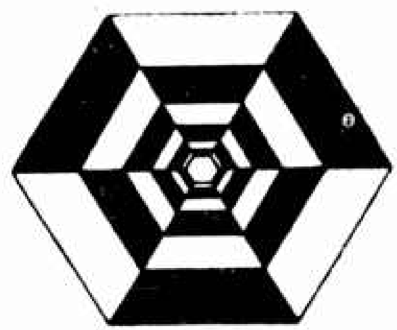

Fig. 18
Similarity Symmetry 3·nM

The author intends to present a more detailed analysis of the operations of similarity symmetry, particularly those applying to three-dimensional figures, at a later date. According to the author, one of the possible fields of application of the new theory could be the problem of rhythmical phenomena in crystallography.

To the writer's knowledge, this article represents the first systematic approach to the little studied problem of similarity symmetry. In addition, the writer believes that the mathematical method of description given by the author could be used for the analysis of periodical and oscillating phenomena not only in crystallography, as mentioned by Shubnikov, but also in designing rotating constructions exposed to rhythmical gravitational action and effects, e.g., interplanetary stations. Thus, the study may have some bearing on space technology.

UNCLASSIFIED

UNCLASSIFIED